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Interference Pattern of X-ray Waves inside the Crystal
and its Image in Vacuum in Double-Field
Approximation (II)

The energy flows of the X-ray waves, diffracted in thick crystals in double-field approximation, have been examined. It has been demonstrated that in a symmetric Laue case the energy flows in the upper layers are oriented in the direction of incidence and reflection, whereas in the lower layers they are oriented in the direction of the normal to the entrance surface of the crystal. The formation of pendular bands during diffraction of X-rays in thick parallelepiped and wedge-shaped crystals has been investigated. A method for obtaining pendular bands from such crystals is described.

Рассмотрены потоки энергии рентгеновских волн, дифрагированных в толстых кристаллах в двухполевом приближении. Показано, что в симметричном случае Лауэ потоки энергии в верхних слоях ориентированы по направлению падения и отражения, а в нижних слоях — по направлению нормали к входной поверхности кристалла. Исследовано образование маятниковых полос при дифракции рентгеновских лучей в толстых параллелепипедальных и клиновидных кристаллах. Описан способ получения маятниковых полос от таких кристаллов.

In a previous communication (ASLANIAN et al.) the interference of all the four Bloch waves in a two-mode case has been investigated. The present paper deals with the problems of the formation and of the observation of the energy flows of all the four Bloch waves and the possibilities of the observation of pendular bands from thick wedge-shaped crystals.

1. Investigation of the flow of energy of diffracted waves in the crystal

The mean value of the total flow of energy of the four waves in a two-mode case is expressed by the formula (James)

\[ S = \frac{c}{8\pi} R(D \times H) \] (1)

where \( R \) means that it is necessary to take only the real part of the expression \( D \times H \); \( D \) and \( H \) are the vectors of the electric field strength and of the magnetic field induction, respectively:

\[ D = D_{01} + D_{02} + D_{h1} + D_{h2} \] (2)

\[ H = (S_{01} \times D_{01}) + (S_{02} \times D_{02}) + (S_{h1} \times D_{h1}) + (S_{h2} \times D_{h2}) \] (3)

where \( S_{01}, S_{02}, S_{h1}, S_{h2} \) are the unit vectors in the directions of the wave vectors \( K_{01}, K_{02}, K_{h1}, K_{h2} \), respectively.
With the help of the expressions (2) and (3) and after some transformations, (1) may be brought to the following form:

\[ S = \frac{c}{8\pi} \frac{1}{2} |D_0|^2 \left\{ \right. \left[ \cos^2 \pi(Ar) + \sin \pi(2H + A) r \sin \pi(Ar) \right] S_{01} + \\
+ \left[ \cos^2 \pi(Ar) + \sin \pi(2H - A) r \sin \pi(Ar) \right] S_{02} + \\
+ \left[ \sin^2 \pi(Ar) + \cos \pi(2H - A) r \cos \pi(Ar) \right] S_{h1} + \\
+ \left[ \sin^2 \pi(Ar) - \cos \pi(2H + A) r \cos \pi(Ar) \right] S_{h2} \left. \right\} \] (4)

Here, by replacing the directions \( S_{01} \) and \( S_{02} \) with the mean direction of the refracted beam \( S_0 \), then \( S_{h1} \) and \( S_{h2} \) with the mean direction of diffraction \( S_h \), we get

\[ S = \frac{c}{8\pi} \frac{1}{2} |D_0|^2 \left\{ \right. \left[ 2 \cos^2 \pi(Ar) + \\
+ \sin 2\pi(Ar) \sin 2\pi(Hr) \right] S_0 + \left[ 2 \sin^2 \pi(Ar) + \\
+ \sin 2\pi(Ar) \sin 2\pi(Hr) \right] \left. \right\} \] (5)

In the expressions (4) and (5) the value \( A_2 A_1 \) is denoted by \( A \).

When the first field is absorbed in the depth of the crystal \( D_{01} = D_{h1} = 0 \) we obtain for the flow of energy from \( 1)-(3) \):

\[ S = \frac{c}{8\pi} \frac{1}{2} |D_0|^2 \left\{ \left( S_{02} + S_{h2} \right) 2 \sin^2 \pi(Hr) \right\} \] (6)

From (4)-(6) we come to the following conclusions:

1. From (4), in the most common case, the energy of the diffracted waves inside the crystal flows along all the directions \( S_{01}, S_{02}, S_{h1} \) and \( S_{h2} \) of the wave vectors taking part in the interference.

2. From (5), disregarding the angles between \( S_{01} \) and \( S_{02} \) and those between \( S_{h1} \) and \( S_{h2} \) the flow of energy of the Bloch waves in the crystal changes its direction, depending on the depth, within the limits from \( S_0 \) to \( S_h \) and only at certain depths, it is directed either to \( S_0 \) or to \( S_h \).

3. From (6), in absorbing crystals only at certain depths, where the first field is completely absorbed, the flow of energy of Bloch waves (of the second field) is directed to the reflecting planes (the energy flows between the reflecting planes).

As a result of these conclusions it may be stated that in the presence of all the four waves, the flow of energy in the crystal is likely to flow in the directions of incidence and of reflection. It is directed to the reflecting planes only after the first field is absorbed.

Consequently, the strong absorption of the first field is apparently due only to the accommodation coefficient (Laue; James).

2. Observation of the flow of energy of diffracted waves outside the crystal

In a previous report (Aslanian et al.) it was pointed out that in a thin crystal all the four waves of the two-mode case take part in the interference and the energy flows in the directions of incidence and of reflection and that in narrow but thick crystals (Fig. 1), where the primary wave falls nearer to the lateral surface, waves having wave vectors \( K^t_{h1} \) and \( K^t_{h2} \) emerge from the upper part of the crystal in the
direction of reflection and form pendular bands. It becomes apparent that from one and the same crystal it is possible to bring out the reflected waves \( K_{h1} \) and \( K_{h2} \) from the lateral surface of the upper part and obtain pendular bands and to bring out the waves of the second field \( K_{02} \) and \( K_{h2} \) from the base as a result of the Borrmann effect (Fig. 1).

We have investigated the distribution of the intensities \( I_2 \) (contrast of the pendular bands) and \( I_1 \) (intensity of the wave \( K_{02} \)) depending upon the distance between the point of incidence of the primary wave or the entrance surface and the lateral surface. For these investigations, we have made use of the sample shown in Figure 1, where the disposition of the wave vectors is visible: those of the primary wave \( K_{01} \) emerging from the lateral surface \( K_{h1} \) and \( K_{h2} \) and those emerging from the base \( K_{03} \) and \( K_{h2} \).

Figure 2a shows the intensities \( I_1 \) and \( I_2 \) depending on the distance between the

**Fig. 1.** Path of rays in a parallelepiped crystal

**Fig. 2.** Intensities of \( I_1 \) and \( I_2 \) depending on the point of incidence of the primary beam (\( AA_1 = d \) — width of crystal; the intensities are indicated in relative units (a), simultaneous recording of all the four beams emerging from the lateral and exit surfaces of the crystal (b).
trace $A_0B_0$ of the incidence of the primary sheet beam and the lateral line $A_1B_1$. In order to distinguish $I_1$ from $I_2$, the base of the sample was polished, then the image of the defects was obtained in the beam $I_4$ and the distribution of the intensity was uniform in the beam $I_2$ (Fig. 2b). The thickness of the samples (in the direction of the reflecting planes (110)) was 12–20 mm, whereas the width (in the direction of the reciprocal lattice vector) was 4–5 mm.

As it is evident from Figure 2a, when the distance between the trace ($A_0B_0$) of the incident beam and the lateral line ($A_1B_1$) is greater than 0.875 mm, the beam $I_2$ disappears and reflected waves do not emerge from the lateral surface. If this distance is reduced, $I_2$ increases and in case when the primary beam is further shifted to the right and partly falls on the crystal on one side, a partial Bragg reflection occurs, which rapidly increases until the primary beam falls completely on the crystal from the side. As it may be seen in this same figure, both in the case of large and small distances of incidence from the lateral surface, the intensity $I_1$ emerges from the crystal with an almost equal intensity. It is interesting to note here that $I_1$ differs from zero even when the primary beam completely falls on the lateral surface. When the primary beam completely falls on the upper part of the lateral surface, then part of the beam is reflected from this surface and beam $I_2$ arises and another part enters the crystal, where a Laue reflection occurs and beam $I_1$ is formed.

It is interesting at this point to pay attention to the following circumstance. We have already noted that when the primary X-ray sheet beam (the width of the slit of the primary beam is 50 µm) falls on the reflecting planes (reflection (220)) at a distance of 0.875 mm and more from the lateral surface, then $I_2$ disappears (a loss of energy of diffracted waves from the lateral surface does not occur). Consequently, we can prepare a narrow (the size of the crystal in the direction of the reciprocal lattice vector is in the direction of the normal of the reflecting planes $\geq 0.875 \times 2$ mm = 1.750 mm) but thick sample (the size of the crystal in the direction of the reflecting planes is about a few centimeters) and create an X-ray diffraction waveguide. In fact, in similar perfect thick crystals, a weakening is practically not observed and the absorption of the second field is insignificantly small after the absorption of the first field.

3. Observation of pendular bands from thick wedge-shaped crystals

In the previous report (Aslanian et al.) we described the observation of pendular bands from a parallelepiped crystal. In similar samples, however, very narrow regions of the crystal, only those situated near the top, take part in the formation of pendular bands. It appears that if we give a wedge-like shape to the sample in such a way that the direction of the thinning lies in the scattering plane parallel to the reflecting planes (Fig. 3), then pendular bands may be formed in the sufficiently broad regions of this wedge-shaped crystal. In ordinary investigations, the thinning of wedge-shaped crystals is directed along the reflecting planes perpendicular to the scattering plane or perpendicular to the reflecting planes; these alternatives do not permit the use of prisms having large refraction angles.

We have prepared wedge-shaped samples from silicon crystals, as it is shown in Figure 3. The reflecting planes (110) are perpendicular to the base of the prism (to the entrance surface) ABCDA and the line AD. One of the reflecting planes is $O_1O_2O_3O_4O_1$. The primary sheet wave falls on the reflecting surfaces in such a manner that its trace on the entrance surface ABCDA is parallel to the central line $O_1O_4$. These samples had approximately the following dimensions: $AD = 5.2$ mm, $O_1O_4 = 12$ mm, $\angle AO_4D \approx 24^\circ$. Because of the wedge-like shape, the paths of the waves having wave
vectors $K_{01}$, $K_{02}$, $K_{h1}$ and $K_{h2}$ were reduced inside the crystal and, therefore, outside the crystal their prolongations were obtained, namely, $K_{h1}^t$ and $K_{h2}^t$ on the right, and $K_{h1}^t$ and $K_{h2}^t$ on the left. When the primary wave fell exactly on the middle of the entrance surface, then all these four waves, superimposed in pairs, were obtained at the same time outside the crystal, which led to the simultaneous formation of pendular bands on the right and on the left (Fig. 4a).

As it is obvious from Figures 4b, c, a shift of the primary beam to the right of the line $O_1O_2$ causes the pendular bands on the left side to disappear (Fig. 4b), whereas a shift to the left makes those on the right side to disappear (Fig. 4c), the nearer the primary beam on the right side is to the lateral line DC, the higher the waves having wave vectors $K_{h1}^t$ and $K_{h2}^t$ emerge (nearer to the line DC) from the crystal and pendular bands are obtained on the left of this beam (Fig. 4d) and the nearer the primary beam falls on the middle line $O_1O_2$, the lower these waves emerge (nearer to the line $O_3O_4$) and pendular bands are obtained on the right (Fig. 4e).

An analogous situation is observed in case when the trace of the incident beam is located on the entrance surface between the lines AB and $O_1O_2$.

We shall examine graphically the emergence of pendular bands in the prism shown in Figure 3. Here we have to bear in mind that the waves diffracted in the direction of the point O of the reciprocal lattice (waves with wave vectors $K_{01}$ and $K_{02}$) emerge from the crystal on the left surface while the waves having wave vectors $K_{h1}$ and $K_{h2}$ emerge from the right surface. Therefore, in order to find the wave vectors $K_{h1}^t$ and $K_{h2}^t$, let us pass through the distribution points $A_1$ and $A_2$ the normals $A_1N_1$ and $A_2N_2$ to the surface $O_4D$, and for finding the wave vectors $K_{01}^t$ and $K_{02}^t$ let us pass through these distribution points the normals $A_1N_3$ and $A_2N_4$. By the intersection of these normals with the spheres of incidence $C_1C_2$ and of reflection $C_3H_4$ we find the points of incidence $P_{01}$, $P_{02}$, $P_{h1}$, and $P_{h2}$, which, when joined with the points O and H of the reciprocal lattice, give us the wave vectors $K_{01}^t$, $K_{02}^t$, $K_{h1}^t$, and $K_{h2}^t$ outside the crystal.

Hence, we find for the periods of pendular bands of the refracted $\Delta_0$ and reflected $\Delta_h$ waves:

$$\Delta_0 = \frac{\sin \theta_0}{|A_1A_2|}$$
$$\Delta_h = \frac{\sin \theta_h}{|A_1A_2|}$$

(7)
where $\theta_0$ and $\theta_h$ are the angles of inclinations of the left and right lateral surfaces $O_4A$ and $O_4D$, respectively (Fig. 5). As it is evident from (7), in case of equal inclinations of the lateral surfaces, the periods $\Delta_0$ and $\Delta_h$ are equal.

4. Conclusions

1. The examination of the general energy flow of Bloch waves shows that it is directed along the normal to the entrance surface and the energy flows along the inter-
planar spaces only after the absorption of the first field (see (6)). In non-absorbing crystals the energy flow is of a complex character (see (4)).

2. A method for obtaining pendular bands from parallelepiped and wedge-shaped crystals having a great angle of refraction has been elaborated. It has been demonstrated that by conveniently choosing the direction of the wedge thinning, it is possible to obtain pendular bands from thick samples having a great angle of inclination. Pendular bands have been obtained from the various sides of the wedge-shaped crystal.

References

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